Mark Scheme 4733
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1	(i)	Method is biased because many pupils cannot be chosen	B1 B1	 **2**	"Biased" or equivalent stated, allow "not random" Valid relevant reason
	(ii)	Allocate a number to each pupil Select using random numbers	B1 B1	 **2**	State "list numbered" Use random numbers [*not* "hat"]
2		$\frac{20-25}{\sigma} = \Phi^{-1}(0.25) = -0.674$ $\sigma = 5 \div 0.674$ $= 7.42$	M1 B1 M1 A1	 **4**	Standardise and equate to Φ^{-1} [*not* .7754 or .5987] z in range [–0.675, –0.674], allow + (±) 5 ÷ z-value [*not* $\Phi(z)$ or 0.75] Answer in range [7.41, 7.42], no sign fudges [SR: σ^2: M1B1M0A0 cc: M1B1M1A0]
3	(a)	Po(1.2) Tables or correct formula used 0.8795	B1 M1 A1	 **3**	Po(1.2) stated or implied Correct method for Poisson probability, allow "1 –" Answer, 0.8795 or 0.879 or 0.88(0)
	(b)	N(30, 30) $\frac{38.5-30}{\sqrt{30}}$ [= 1.55] [$\Phi(1.55)$ =] 0.9396	B1 B1 M1 A1 A1	 **5**	Normal, mean 30 stated or implied Variance 30 stated or implied, allow √30 or 30^2 Standardise using $\sigma^2 = \mu$, allow √ or cc errors √μ and 38.5 both correct Answer in range [0.939, 0.94(0)]
4	(i)	$\hat{\sigma}^2 = \frac{50}{49} \times 0.0967 = 0.0987$	M1 A1	 **2**	Use $\frac{n}{n-1} \times s$ or s^2, allow √ Answer, a.r.t. 0.0987
	(ii)	$H_0: \mu = 1.8$, $H_1: \mu \neq 1.8$ where μ is the population mean	B1B1		Hypotheses correctly stated in terms of μ SR: μ wrong/omitted: B1 both, but $\overline{X}$: B0
	α, β:	$z = \frac{(1.72-1.8)}{\hat{\sigma}/\sqrt{50}} = -1.8(006)$	M1 A1		Standardise with √n, allow +, biased σ, √ errors $z = -1.80 \pm 0.01$, don't allow +
	α:	$-1.8 < -1.645$	B1√		Compare ± z with ±1.645, signs consistent
	β:	$\Phi(-1.8) = 1 - 0.9641 < 0.05$	B1		Explicitly compare $\Phi(z)$ with 0.05, correct tail
	γ:	CV $1.8 - k.\sigma/\sqrt{50}$ $k = 1.645$, CV = 1.727 1.72 < 1.727	M1 A1√ B1√		Correct expression for CV, – or ±, k from Φ^{-1} CV = 1.727, √ on their k, ignore upper limit $k = 1.645$ and compare CV with 1.72
	Reject H_0 Significant evidence that mean height is not 1.8		M1 A1√	 **7**	Reject H_0√, correct method, needs √50, μ = 1.8; allow cc,√ or k error or biased σ estimate Conclusion stated in context [SR: 1.8, 1.72 interchanged: B0B0M1A0B1M0]
5	(i)	${}^{30}C_{10}(0.4)^{10}(0.6)^{20}$ or 0.2915 – 0.1763 = 0.1152	M1 A1	 **2**	Correct formula or use of tables Answer, a.r.t. 0.115
	(ii)	$30p > 5$ so $p > \frac{1}{6}$ $30q > 5$ so $q > \frac{1}{6}$ $\frac{1}{6} < p < \frac{5}{6}$	M1 M1 A1	 **3**	$30p$ or $30pq$ used $30q$ or both solutions from $30pq$ used *Either* $\frac{1}{6} < p < \frac{5}{6}$ *or* $\left[\frac{1}{2} - \frac{\sqrt{3}}{6} < p < \frac{1}{2} + \frac{\sqrt{3}}{6}\right]$ [0.211 < p < 0.789], allow ≤
	(iii)	N(12, 7.2) $\frac{10.5-np}{\sqrt{npq}}$ and $\frac{9.5-np}{\sqrt{npq}}$ $\Phi(-0.559) - \Phi(-0.9317)$ = 0.8243 – 0.7119 = 0.1124	B1 B1 M1 A1√ M1 A1	 **6**	12 seen 7.2 or 2.683 seen, allow 7.2^2 Both standardised, allow wrong/no cc, npq √npq, 10.5 and 9.5 correct, √ on their np, npq Correct use of tails Answer, in range [0.112, 0.113] [SR: $\frac{1}{\sqrt{2\pi \times 7.2}} e^{-\frac{1}{2}\frac{(10-12)^2}{7.2}}$ M1A1, answer A2]

6	(i)	$R \sim B(25, 0.8)$ $P(R \le 16) = 0.0468$, $P(R \le 17) = 0.1091$ $k = 16$	B1 M1 A1	 **3**	B(25, 0.8) stated or implied, e.g. from N(20, 4) One relevant probability seen [Normal: M0A0] Answer $k = 16$ only [SR: unsupported 16, B1M0B1]
	(ii)	$20p$ $= 0.936$	M1 A1	 **2**	$20 \times$ their p *or* 20×0.05 Answer, a.r.t. 0.936, i.s.w.
	(iii)	$P(R \le 16 \mid p = 0.6)$ $= 0.7265$	M1 A1	 **2**	Find $P(R \le k \mid p = 0.6)$ Answer 0.7265 or 0.727
	(iv) α:	$p' = 0.5 \times 0.0468 + 0.5 \times 0.7265$ $= 0.38665$ $2 \times p' \times (1 - p')$ $= 0.474$	M1 A1 M1 A1	 **4**	"Tree diagram" probability, any sensible p Value in range [0.38, 0.39] Correct formula, including 2, any p' Answer in range [0.47, 0.48]
or β:		0.8 A 0.8 R $.5^2 \times .9532 \times .0468 = .0112$ 0.8 R 0.8 A $.5^2 \times .0468 \times .9532 = .0112$ 0.6 A 0.8 R $.5^2 \times .2735 \times .0468 = .0032$ 0.6 R 0.8 A $.5^2 \times .7265 \times .9532 = .1731$ 0.8 A 0.6 R $.5^2 \times .9532 \times .7265 = .1731$ 0.8 R 0.6 A $.5^2 \times .0468 \times .2735 = .0032$ 0.6 A 0.6 R $.5^2 \times .2735 \times .7265 = .0497$ 0.6 R 0.6 A $.5^2 \times .7265 \times .2735 = .0497$	M1 A1 M1 A1		$p_1q_2 + p_2q_1$ etc (0.5 not needed) 4 cases, √ on their ps and qs, 0.5 not needed e.g. $2(p_1q_2 + p_2q_1)$ Completely correct list of cases and probabilities, including 0.5 Answer in range [0.47, 0.48]
7	(i)	$(11 - 3)k = 1$ $k = 1/8$	M1 A1	 **2**	Use area = 1 [e.g. $\int kx\,dx = 1$ with limits 3, 11] Answer 1/8 or 0.125 only
	(ii)	$\mu = \frac{1}{2}(3 + 11) = 7$ $\int_3^{11} \frac{1}{8}x^2\,dx = \left[\frac{x^3}{24}\right]_3^{11}$ $[= 54\frac{1}{3}]$ $\sigma^2 = 54\frac{1}{3} - 7^2$ $= 5\frac{1}{3}$	B1 M1 A1 M1 A1	 **5**	Mean 7, cwd Attempt $\int x^2 f(x)\,dx$, correct limits Indefinite integral $\frac{x^3}{3k}$, their k Subtract their μ^2 Correct answer, $5\frac{1}{3}$ or a.r.t. 5.33
	(iii)	$P(X < 9) = 6k$ $[= \frac{3}{4}]$ $(\frac{3}{4})^3$ $= \frac{27}{64}$ or 0.421875	B1√ M1 A1	 **3**	Correct p for their k Work out their p^3, $0 < p < 1$ Answer $\frac{27}{64}$ or a.r.t. 0.422
	(iv)	Normal Mean is 7 Variance is $5\frac{1}{3} \div 32$ $(= \frac{1}{6})$	B1 B1√ B1√	 **3**	"Normal" distribution stated Mean same as in (ii) √ Variance is [(iii) ÷ 32] √ [*not* √ errors]
8	(i)	Coins occur at constant average rate and independently of one another	B1 B1	 **2**	One contextualised condition, e.g. independent A different one, e.g. constant average rate, or "not in hoards" ["singly" not enough]. Treat "random" as equivalent to "independent". Allow "They…"
	(ii)	$R \sim Po(5.4)$ $e^{-5.4}\frac{5.4^3}{3!} = 0.1185$	B1 M1 A1	 **3**	Poisson (5.4) stated or implied Correct formula, any λ Answer, in range [0.118, 0.119]
	(iii)	$R \sim Po(3)$ Tables, looking for 0.05 or 0.95 $P(R \ge 7) = 0.0335$ Therefore smallest number is 7	B1 M1 A1√ A1	 **4**	Poisson (3) stated or implied Evidence of correct use of tables One relevant correct probability seen $r = 7$ only, ignore inequalities
	(iv)	$R \sim Po(4.8)$ Type II error is $R < 7$ when $\mu = 4.8$ $P(< 7) = 0.7908$	B1 M1 A1	 **3**	Poisson (4.8) used Correct context for Type II error, √ on their r P(< 7), a.r.t. 0.791, c.w.o. [P(≥ 7): M0]